Math 112 review questions for the first midterm

The midterm covers everything* we have done in class, tutorial and hand-in homework up until Friday, October 14. Orders ($<$) are not on the midterm. I will give you a sheet of the axioms A1 - A4, M1 - M4, C and D.

(* everything except orders)

(1) Make a list of the important topics that we have covered so far, and the types of questions that you have seen done in class and in homework.

(2) Define the following terms (and be able to give an example if appropriate): set, subset, powerset, intersection, union, cartesian product, function, commutative, associative, additive/multiplicative identity, additive/multiplicative inverse, binary relation, closed, axiom, abelian group, commutative ring, integral domain, zero divisor.

(3) What is Russel's paradox? What set did he define and why was this a problem?

(4) In a paragraph, argue either for or against the process of writing down axioms for arithmetic.

(5) Do *all* of the practice problems in chapters 1 and 2. There are solutions to these at the end of the chapter, so you can check your work.

(6) The following questions (numbered ones are from the Exercises at the end of each chapter) are also good practice. Those marked with * were homework.
   - 1.3*, 1.4*, 1.5*
   - What must be true about the sets A and B in order that $A \times B = B \times A$? Justify your answer
   - 1.7, 1.8, 1.9*, 1.11*, 1.13*, 1.14, 1.17*, 1.18, 1.21, 1.24*, 1.30
   - Solve for x in one's digit arithmetic, using axioms one step at a time: $3x + 1 = 0$. Is there more than one x that is a solution?
   - Is there more than one number x in one's digit arithemetic that satisfies $2x + 2 = 2$? Why?
   - 2.1, 2.2*, 2.7*
   - Prove using only the axioms that for any elements a and b in an abelian group, if $a + (-b) = (-b) + c$, then $a = c$
   - Suppose a, b, c and d are elements of a commutative ring. Prove that $(a \cdot b^{-1}) + (c \cdot d^{-1}) = ((a \cdot d) + (b \cdot d)) \cdot (b^{-1} \cdot d^{-1})$ [note: this is a more lengthly proof than you should expect on the exam, but it is probably good practice to do this one, or at least figure out what all the steps should be]